CORRESPONDENCE

Vertical Displacements of Air Parcels During Strong Convergence and Heavy Precipitation

KENDALL R. PETERSON

Hydrometeorological Section, U. S. Weather Bureau, Washington, D. C.

May 22, 1957

Meteorologists may be interested in the large vertical displacements of air that take place during heavy rainfall. The height to which the parcels rise during an interval of time, given the distribution of horizontal velocity divergence, has been investigated as part of an analytical study of precipitation processes. With the nomogram developed by Peterson [1] this vertical displacement can be related to precipitation.

The relationship between vertical velocity and horizontal velocity divergence, assuming hydrostatic equilibrium, is defined by the continuity equation expressed in the form

$$\frac{\partial \omega}{\partial p} = -\text{Div } \mathbf{V} \tag{1}$$

where $\omega \equiv \frac{dp}{dt}$ is the vertical velocity for a pressure vertical coordinate. This equation can be integrated for a given variation of divergence with pressure. The variation is usually determined by computing the divergence for several constant pressure levels. It will be assumed here that the vertical distribution of divergence during cyclonic activity is a linear one, such that,

$$\text{Div } \mathbf{V} = -(b + cp) \tag{2}$$

where b and c are arbitrary constants that can be evaluated by assuming values of divergence at two levels.

Substitution of equation (2) into equation (1), and integration give

$$\frac{dp}{dt} = a + bp + \frac{1}{2} cp^2 \tag{3}$$

where a is an integration constant that can be evaluated by assuming a value for $\frac{dp}{dt}$ at the surface.

Integration of equation (3) over the time interval Δt gives

$$\left(\frac{cp + b - \sqrt{b^2 - 2ac}}{cp + b + \sqrt{b^2 - 2ac}}\right)_f = \left(\frac{cp + b - \sqrt{b^2 - 2ac}}{cp + b + \sqrt{b^2 - 2ac}}\right)_i \exp\left(\Delta t \sqrt{b^2 - 2ac}\right) \tag{4}$$

where the subscripts f and i refer to the final and initial pressure of the air parcel and t is the time required for the parcel to ascend from p_i to p_f. If it is assumed that the level of non-divergence is at 600 mb. and that the vertical

TABLE 1.—*Vertical displacement of rising air parcels associated with 1000-mb. convergence*

Initial pressure on air parcels (mb.)	For Div V_0= −0.036 hr.$^{-1}$ vertical displacement (mb.) during			For Div V_0= −0.36 hr.$^{-1}$ vertical displacement (mb.) during			For Div V_0= −3.6 hr.$^{-1}$ vertical displacement (mb.) during	
	1 hr.	3 hr.	6 hr.	1 hr.	3 hr.	6 hr.	1 hr.	3 hr.
200	0	0	0	0	0	0	0	0
300	3	9	18	28	63	87	97	100
400	6	15	30	48	119	170	193	200
500	8	20	40	65	163	250	287	300
600	10	20	43	70	198	316	378	400
700	10	19	43	70	211	368	465	500
800	6	17	34	60	194	395	539	600
900	3	9	20	39	137	342	570	700
1000	0	0	0	0	0	0	0	0

TABLE 2.—*Vertical displacement (mb) for one-half inch, one inch, and five inches of precipitation for saturated pseudo-adiabatic atmosphere with various surface dewpoints*

Initial pressure of air parcel (mb)	Surface dewpoint								
	40° F			60° F			75° F		
	0.5 in.	1 in.	5 in.	0.5 in.	1 in.	5 in.	0.5 in.	1 in.	5 in.
200	0	0	0	0	0	0	0	0	0
300	85	98	100	59	84	100	46	72	100
400	166	196	200	112	164	200	83	137	200
500	241	292	300	154	237	300	112	193	299
600	305	387	400	183	300	400	129	237	399
700	349	479	500	193	346	500	131	261	498
800	362	562	600	177	360	600	115	252	597
900	310	616	700	121	299	699	73	185	693
1000	0	0	0	0	0	0	0	0	0

velocity at 1000 mb. is zero, equation (4) reduces to

$$\left(\frac{200 - p_f}{1000 - p_f}\right) = \left(\frac{200 - p_i}{1000 - p_i}\right) \exp\left(\Delta t \text{ Div } \mathbf{V}_0\right) \tag{5}$$

where Div $\mathbf{V}_0$ is the divergence at 1000 mb.

A graphical solution to equation (5) is given in figure 1. The extreme left scale is the 1000-mb. divergence. Both positive and negative values are included, thereby enabling the use of the nomogram for computation of upward or downward vertical displacement. The horizontal line is the time (Δt) scale. The vertical scales to the right are a reference line (R) (solving the product Δt Div $\mathbf{V}_0$), the final pressure (p_f), and the initial pressure (p_i). Table 1 was prepared from equation (5) and indicates pressure displacements of air parcels at 100-mb. increments.
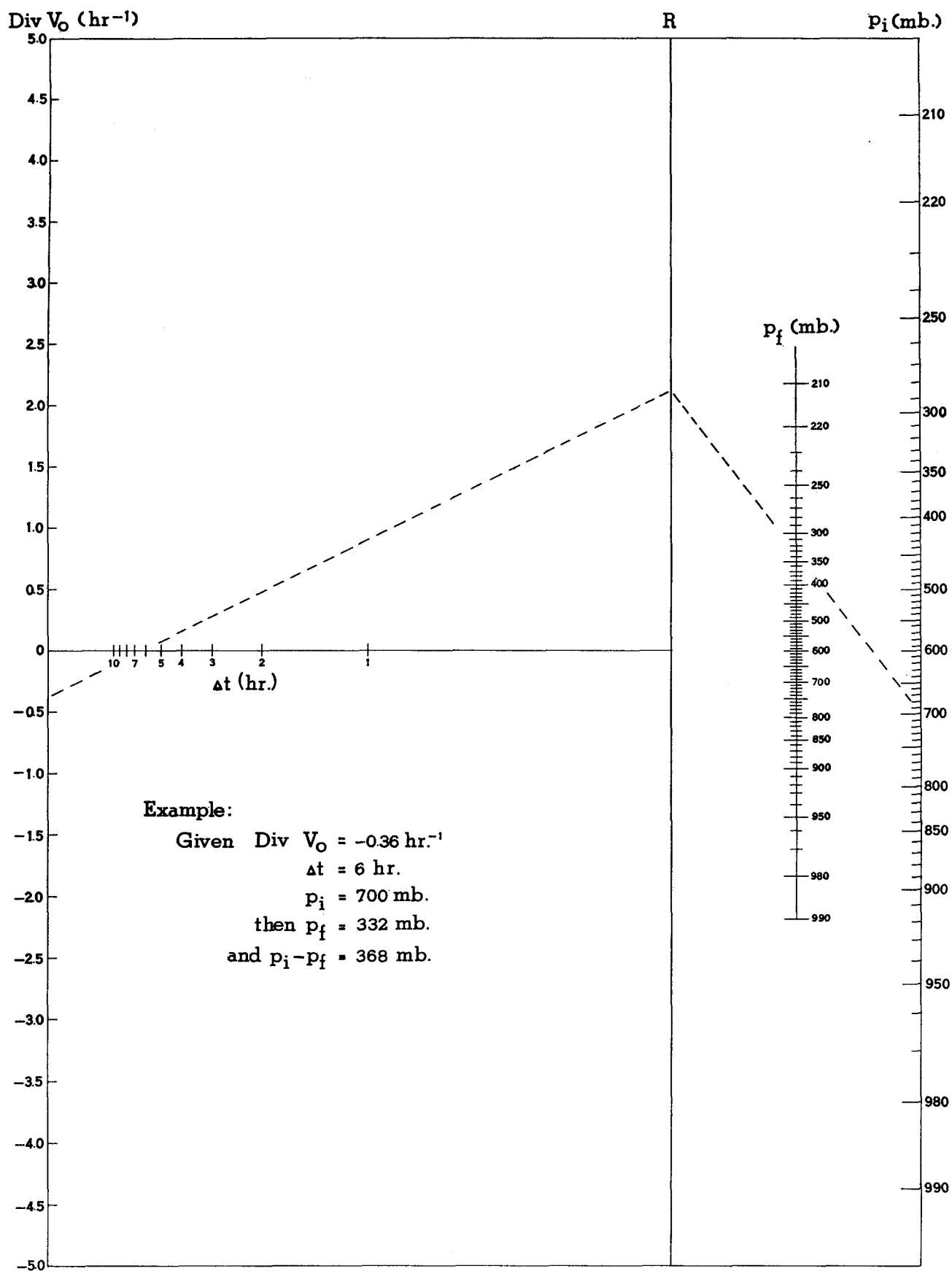
FIGURE 1.—Nomogram for computing vertical displacement (final pressure p_f) as a function of initial pressure (p_i), horizontal divergence at 1000 mb. (Div V_0), and time (Δt). The nomogram is based on the assumption of linear change of horizontal divergence with pressure, with zero divergence at 600 mb.

Under the assumptions made here and in the previous paper [1] (all of which seem realistic), the vertical displacement is related directly to the precipitation that falls from a column and to the moisture conditions in the column (regardless of the time during which the precipitation occurs). Table 2, computed from these assumptions, gives typical values of vertical pressure displacement for rainfall amounts of 0.5 inch, 1.0 inch and 5 inches.

The writer wishes to thank Dr. C. S. Gilman for suggesting the idea which is an elaboration of pages 57–61 of reference [2].

REFERENCES

1. K. R. Peterson, "Precipitation Rate as a Function of Horizontal Divergence," *Monthly Weather Review*, vol. 85, No. 1, January 1957, pp. 9–10.
2. C. S. Gilman, An Expansion of the Thermal Theory of Pressure Changes, Doctoral Dissertation, Department of Meteorology, M. I. T., 1949 (unpublished).

Weather Notes

versity of California at Los Angeles (Weather Bureau Contract CWB 7904), June 1950; see Part I, p. 16). These parallel bands with associated bright spots may offer good opportunity for estimating cloud movements as the bands appear to be a fairly permanent feature of the cloud mass. Unfortunately, no theodolite readings were taken on these parallel bands.

At the time the clouds were first observed, none of the forecasters or observers on duty had ever seen noctilucent clouds (Mr. Glommen came on duty at 0100 AST). Since the clouds were on the north-northeastern horizon near the sun's rays at this time of year, a few people believed them to be very high cirrus such as have been reported by jet aircraft pilots at altitudes above 40,000 or 45,000 ft. However, after identifying cirrus clouds the following day and observing the behavior of the sunlight on these clouds after sunset, no doubt remained that the clouds observed the previous night were at vastly higher elevations than normal cirrus clouds. The cirrus clouds observed on Sunday night were entirely dark by 2300 AST. The noctilucent clouds seen Saturday night were definitely in the sun's rays and had no significant color changes from the time they were first observed until they disappeared near sunrise. A faint orange glow caused by smoke or haze persisted in the northern sky through the night.—*W. B. Lindley, Meteorologist in Charge, WBAS, Anchorage, Alaska.*

TEMPERATURE AND WIND FIELDS AT THE TIME OF NOCTILUCENT CLOUDS IN ALASKA, JULY 27–28, 1957

Most noctilucent clouds are observed at elevations of 65 to 90 km. in the vicinity of the mesopause between ozonosphere and ionosphere. Although these clouds occur well above the highest layers from which radiosonde data are obtained, an examination of constant pressure charts was made.

The wind flow at 50 and 25 mb. over southern Alaska from 24 hours before to 24 hours after the observation of noctilucent clouds described in Mr. Lindley's note was generally easterly, 5–10 knots. Winds and height changes indicate that a weak trough moved across the area from west to east during the period. At 500 mb., a stronger trough in the westerly winds simultaneously moved in the same direction, suggesting that the trough effect was impressed from below the 50-mb. level.

The temperature field was very weak and typical of that month. Temperatures at both 50 and 25 mb. were warmer to the north so that the easterly winds increased upward through the layer and also for some distance above the 25-mb. level.

At sea level and 500 mb. a weak residual Low moved into the sea area south of Anchorage with little wind flow over the mountain ranges.

A possible explanation of the noctilucent clouds can be found in the apparent general convergence in the vicinity of the trough in a deep layer extending above the 50-mb. level. This is suggested by temperature decreases of one or two degrees at 50 mb. and of about one degree at 25 mb. during a period when cold air advection was not indicated by the streamline-isotherm pattern.

In summation, there is only a slight and inconclusive indication of conditions at 50 and 25 mb. that might explain an unusual event such as noctilucent clouds at a height of possibly 80 km.—*S. Teweles, U. S. Weather Bureau, Washington, D. C.*

RARE WATERSPOUTS IN ALASKA

On August 19, 1957 between 0900 and 1045 PST, two distinct waterspouts were observed in Cross Sound, 20 miles southwest of Cape Spencer Light Station, Alaska. These were sighted by Goody Winthrop, a deep-sea fisherman. During this period, he also observed several other waterspouts in the process of forming. These were observed to start downward from a cloud formation which he estimated to be 1,000 feet high.

On that day Cape Spencer reported an estimated 1,500-foot overcast, visibility 15 miles, temperature 51° F., dewpoint 49° F., and the wind east-southeast at 12 knots. The synoptic situation showed a cold trough over the eastern Gulf of Alaska with a cold cut-off Low over the southeastern gulf. The nearest raob report, taken at Yakutat, 150 miles northwest of Cross Sound, at 0400 PST showed moist unstable air from the 900-mb. level to 600 mb. with an isothermal layer from the surface to 900 mb. The average lapse rate slightly exceeded 4° F. per 1,000 feet and the degree of instability for that stratum was about −1.6° C. (difference between temperature of air parcel after being lifted from 900 to 600 mb. and the observed 600-mb. temperature). Heavy rain showers were reported throughout the area by pilots, and lightning was also reported in Juneau the previous night, which lends support to the statement that the air was unstable.

Another interesting fact was the abnormally high sea temperatures in the eastern part of the gulf. The FWS Research Vessel *Cobb* measured sea temperatures of 64° F. between Cape Ommaney and Cape St. Elias during that period on the 19th although in the Cross Sound area itself the reading was 54° F. It could not be ascertained for sure whether or not the waterspouts appeared where the sea temperatures were abnormally high.—*Gordon D. Kilday, WBAS, Juneau, Alaska.*